Passive Radiation Imaging

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Introduction

This project examined the mathematical feasibility of inferring the internal geometry of an object based on passive measurements of emitted radiation.

- The measured data is radiation emitted from the object.
- Based on this data we attempt to reconstruct the attenuation and radiation source density within the object simultaneously.
- We assume that there is no scattering of radiation within the object.

The work is motivated by the need for a passive technique to determine the internal structure of devices containing radioactive material for the purpose of arms treaty enforcement.

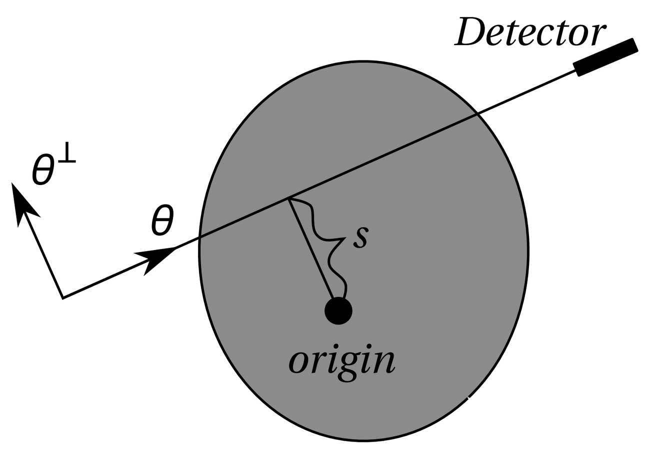

Figure 1: Radiation emitted along a given line is attenuated by an object. We parametrize lines by the co-ordinates (s, θ).

The forward problem

We model the emitted radiation using the attenuated Radon Transform (AtRT).

- The attenuation within the object is a.
- The source density within the object is f.

The AtRT is

$$R_a f(s,\theta) = \int_{-\infty}^{\infty} f(s\theta^{\perp} + t\theta) e^{Da(s\theta^{\perp}+t\theta,\theta)} \, \mathrm{d}t, \tag{1}$$

where Da is the Beam transform given by

$$Da(x,\theta) = \int_0^{\infty} a(x + t\theta) \, \mathrm{d}t.$$

Previous research

- Research in [1] shows non-uniqueness in the recovery a and f, requiring a good initial guess to be made in order to recover correct maps. Explicit pairs of radial a and f which give the same AtRT are given.
- In some cases stable recovery is possible. Research in [2] does provide criteria for the stable recovery of a and f.
- The inclusion of first order scattering, as well as ballistic, emission data studied in [3] leads to an improved reconstruction.

Our method

We take the AtRT given in (1) and discretize to produce a matrix problem. Then a gradient descent method is used to solve the resulting inverse problem. The discretization is performed in the following way (see Figure 2):

- The area of interest is split into a grid of pixels made up of horizontal and vertical lines spaced dx apart.
- The pixels are labelled lexicographically from top left to bottom right.
- We assume that both a and f are piecewise constant over each of the pixels.
- Choose a line from the emission data and record the pixels it passes through and the order in which they are passed.
- Record the intersection points of a chosen ray with the grid in an ordered set K and the distances between each of the intersection points in an ordered set IT.

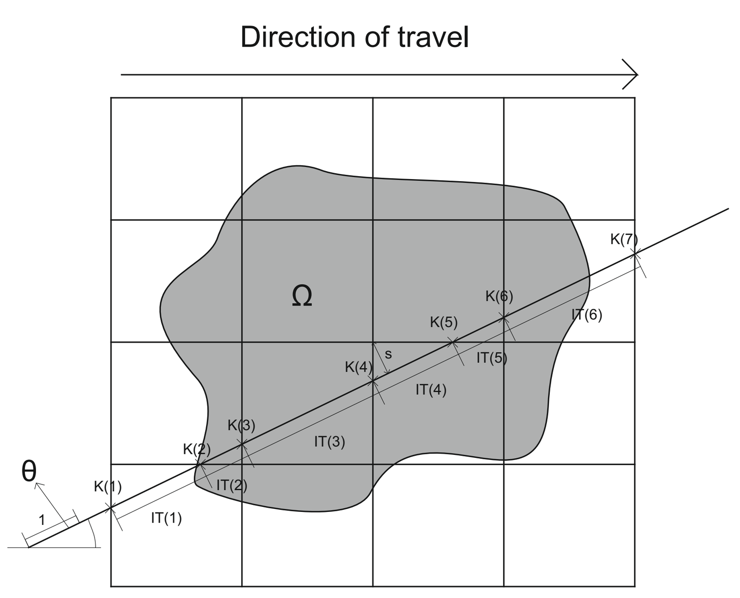

Figure 2: Example of discretization. Here the ray passes through the pixels 13,9,10,11,7,8

- The discretized transform is then in the form of a matrix RAD_a which depends on the attenuation a. Given emission data d we apply gradient descent to the objective function,

$$\mathcal{J} = \|RAD_a f - d\|_2^2 \tag{2}$$

- The derivatives of $\mathcal{J}$ are taken with respect to a and f to give search directions for gradient descent.
- We begin with an initial guess for a and then compute the initial f using least squares.
- The method alternates between iterations modifying f and a.

Numerical examples

In the following reconstructions using synthetic data, the top row gives a and the bottom f. The 1st column is the actual a and f using to create the synthetic data. The second column is computed with an initial guess $a = 0$ and the third $a = 0.5$. The last example (Figure 5) is computed with an exact f.

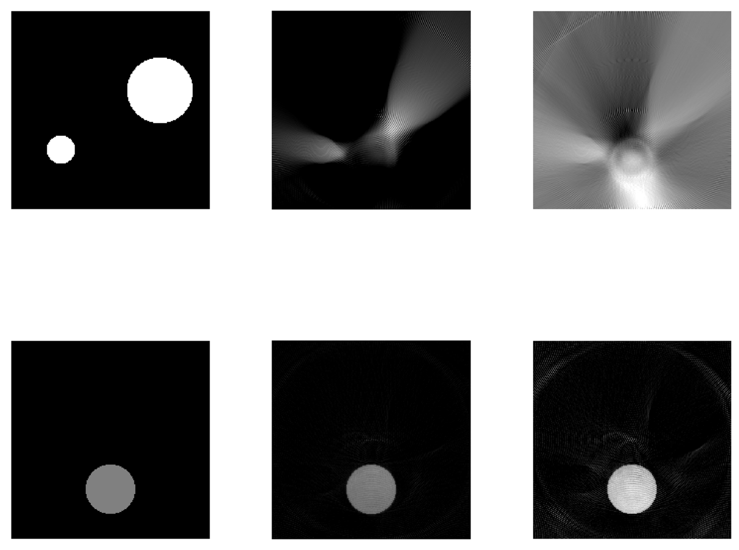

Figure 3: This demonstrates uniqueness issues and cross-talk effects depending on the initial choice of a. There are artifacts here which may be reduced by the addition of regularization.

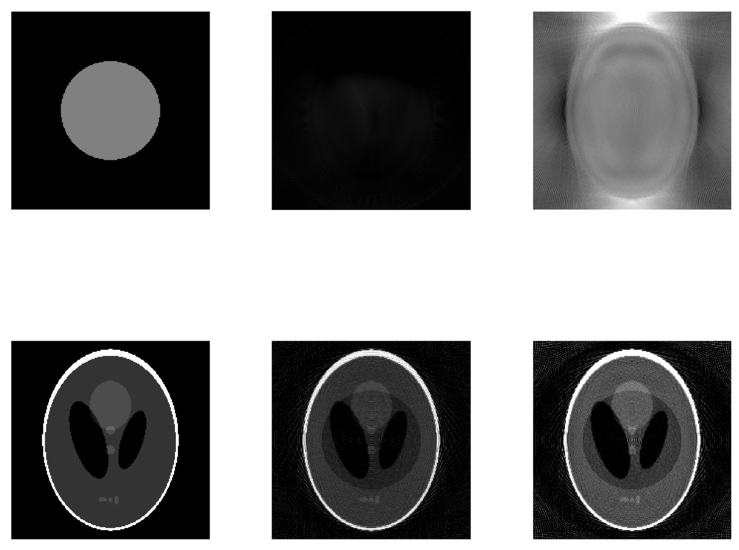

Figure 4: This shows the cross-talk phenomenon again and the non-uniqueness in the recovery, in this case picking the background attenuation as the inital guess leads to no a recovery at all.

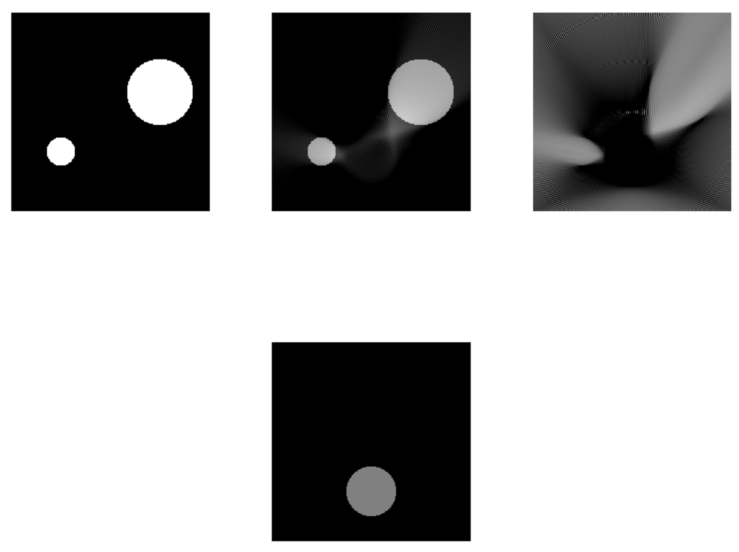

Figure 5: This is another example which stresses the importance of making a good initial guess, here the best reconstruction occurs when the intial guess is taken as the background a in contrast to the previous examples.

Conclusion

We have found that without some additional hypothesis on a and f, reconstructions are likely to contain artifacts including cross-talk. Possible extensions to this project include:

- The inclusion of regularization to the described method. In particular we would like to incorporate the requirement that the attenuation can only take certain values.
- Construction of the matrix RAD_a is trivially parallizable, and so coding it for a GPU is likely to make the production of 3D images much faster.

Acknowledgment

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References

[1] D. Gourion and D. Noll, *The inverse problem of emission tomography*, Inverse Problems **18**, 2002, pp. 1435-1460.

[2] S. Luo, J. Qian, and P. Stefanov, *Adjoint State Method for the Identification Problem in SPECT: Recovery of both the Source and Attenuation in the Attenuated X-Ray Transform*, SIAM J. Imaging Sciences **7**(2), 2014, pp. 696-715.

[3] M. Courdurier, F. Monard, A. Osses, and F. Romero, *Simultaneous source and attenuation reconstruction in SPECT using ballistic and single scattering data*, Inverse Problems **31**(9), 2015, 095002 (30pp).